Lab 12 - The Gamma Function

For this lab we will be considering the following function:

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx \quad \alpha > 0$$

1. Find $\Gamma(1)$ by hand.

2. Find $\Gamma(2)$ by hand using integration by parts. (*Hint*: $\lim_{x \to \infty} x^n e^{-x} = 0$ for all n)

3. Use Maple to find $\Gamma(3)$, $\Gamma(4)$, $\Gamma(5)$, and $\Gamma(6)$.

4. Conjecture about the value of $\Gamma(\alpha)$ for any integer α. Show how this implies that $0! = 1$.

5. Assume $\alpha > 1$, though not necessarily an integer. Show that $\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$ by using integration parts and the hint from above.

6. Show how this could be used to develop a general factorial definition: $\alpha! = \Gamma(\alpha + 1)$.

7. Use Maple to find $\Gamma\left(\frac{1}{2}\right)$, then use it to find: $(.5)!$, $(1.5)!$, $(2.5)!$, and $(3.5)!$